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Innovative Applications of O.R.

Sustainable vegetable crop supply problem

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ABSTRACT

We consider an agricultural production problem, in which one must meet a known demand of crops while respecting ecologically-based production constraints. The problem is twofold: in order to meet the demand, one must determine the division of the available heterogeneous arable areas in plots and, for each plot, obtain an appropriate crop rotation schedule. Rotation plans must respect ecologically-based constraints such as the interdiction of certain crop successions, and the regular insertion of fallows and green manures. We propose a linear formulation for this problem, in which each variable is associated with a crop rotation schedule. The model may include a large number of variables and it is, therefore, solved by means of a column-generation approach. We also discuss some extensions to the model, in order to incorporate additional characteristics found in field conditions. A set of computational tests using instances based on real-world data confirms the efficacy of the proposed methodology.

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1. Introduction

The conventional agricultural production and distribution model is based mainly on large monocultures, and on an intensive use of capital, pesticides, synthetic fertilizers and other non-renewable and polluting resources. Although this model allows for a reduction in food production costs, there are important and usually non-computed side-effects such as the environmental costs of water contamination by pesticides and other polluting inputs or the social costs associated with the exclusion of small farmers due to the increasing requirements of capital (Altieri, 1995; Gliessman, 2000). These side-effects which until very recently did not influence farmers or societal choices concerning production methods threaten the sustainability of these food production models and highlight the need for new agricultural paradigms (Tillman et al., 2002).

In view of the above, more sustainable agricultural production and distribution models have recently gained attention. Indeed, an increasing number of consumers and governments are becoming more conscious about these topics and there is now an extra monetary value associated with products originating from more ecologically-based and social-friendly agricultural systems, such as organic, biodynamical, equitable, fair-trade systems, etc. (Makatouni, 2002; Seyfang, 2006; Lyon, 2006). This premium value has propelled the development of these alternative models, giving rise to several new planning problems in which one must consider both technical and ecological production aspects, as well as the access of small farmers to the market. On the production side, for example, one is now concerned with the diversity of the production fields, the preservation of the productive resources, and the effective use of these resources in a local or regional level and the adoption of cultural and biological methods in order to control the population of weeds, herbivores and pathogens. On the distribution side, one must ensure that small farmers can cost-effectively offer their productions to consumers and that the generated benefits are distributed fairly among the supply-chain participants.

In this article, we consider a situation commonly faced by small family farmers of vegetable crops in Brazil. These producers own small cropping areas yielding small and discontinuous productions. This discontinuity is due mainly to climatic conditions which restrict the growth of some crops in certain areas and periods. The small production volumes do not allow investments in cleaning, packing and distribution structures; on the other hand, the discontinuous aspect of production makes difficult the establishment of stable links between vegetable farmers and markets since the latter requires a continuous supply of products.

In face of these problems, some small vegetable farmers unite in collective organizations such as cooperatives or associations. These organizations promote the gathering of many small productions in packing houses for standardization and distribution, according to long term contracts with consumers and supermarkets. In order to achieve this, these associations must organize the size of areas to be grown with each crop in order to meet demands, and decide where and when this production will occur, due to the fact that each small producer's land might be located in different climatic regions and, therefore, be subjected to different production constraints. In the case of ecologically-based agriculture, additional

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technical–ecological constraints must be considered. Among these constraints, one can cite (a) the undesirability of growing two crops of the same botanic family in sequence on the same piece of land in order to reduce the propagation of pests, (b) the periodical growth of the so-called green manure, usually some legume species, which help protect the soil and increase its fertility, and (c) the use of fallows, when the natural vegetation is left to grow in order to help reestablish the structure and fauna of the soil and reduce pest damage (Altieri, 1995; Gliessman, 2000). Criteria (a)–(c) suggest the problem of determining in which order crops should be grown on a piece of land. A crop rotation plan is a solution for this problem and will be studied in Section 2.

In view of the above description, we define the *sustainable vegetable crop supply problem* (SVCSP), which can be described as the problem of determining the best division of various heterogeneous pieces of land in order to meet a known demand and optimize a given metric, such as maximize production volume or revenues. Moreover, the crop rotation plan on each piece of land must respect constraints (a)–(c). Note that the complexity of this problem is increased by the fact that it concerns vegetable crop growers. These growers usually cultivate a large number of crop species in side-by-side plots. These vegetable species belonging to different botanic families, present very different production times and for the most part, have planting seasons determined by climate conditions.

The problem of deciding the optimal distribution of arable land is certainly not new to operation researchers. Indeed, as early as 1939, Kantorovich had already mentioned this problem as one that could be dealt with by mathematical tools (Kantorovich, 1960 – translated from the Russian original, dated 1939). The author proposes a mathematical model that selects the optimal partition of land areas in order to maximize the production while the proportion of the different crop production respects a given plan. Hildreth and Reiter (1951), in one of the first conferences on applications of operations research, state that crop rotation can be beneficial since it can increase crop yields (due to the fact that distinguished rotations produce different effects on the soil). Moreover, the authors mention that dividing the available land among different rotations might reduce the need for resources (such as water, labor, etc.) since different crops can have different needs throughout the year, and provide some sort of security against failure of a given crop in one year. The authors use a list of pre-determined rotations, which simplify the problem.

El-Nazer and McCarl (1986) eliminate the need for pre-determined rotations with the assumption that the yield of a crop only depends upon the crops grown in the same land in the previous years. Haneveld and Stegeman (2005) use the idea of crop succession requirements, which are expressed in terms of infeasible sequence of crops. The proposed model allows the use of a solution method based on a max-flow algorithm. Detlefsen and Jensen (2007) take into account the fact that crop rotation influences the needs of nitrogen and the yield of the fields. Their model considers that the amount of land to be planted with each crop is given, for each year, which enables the formulation of the crop rotation problem as a transportation problem. In this and in the previous mentioned work, the crop production times are assumed to be 1 year, the available cropping area is homogeneous and the authors explicitly forbid infeasible crop sequences. Clarke (1989) considers crops with different production times and planting seasons of the year. However, there are no constraints in the sequence of crops.

Other work in the literature has presented decision-aid support systems that evaluate the effect of diverse rotations (Dogliotti et al., 2003; Jones et al., 2003; Stöckle et al., 2003; Bachinger and Zander, 2006). In this work, the computational implementations usually serve as aid tools for agricultural specialists, which can provide a given crop rotation and obtain the effects on yields and soil conditions from the tool.

The short review presented above exemplifies the richness of the research in this area. Indeed, the same basic situation might originate various distinguished problems, depending on the particular objectives or constraints that are considered. In the practical problem that motivated this study, one can highlight the need of meeting a known demand from production coming from heterogeneous pieces of land, while respecting some ecologically-based agricultural constraints. These main particularities can be summarized in the following five characteristics:

1. A known demand for each crop (associated with the existing contracts with consumers, for example) should be met.
2. Each crop has an earliest and a latest planting time that must be respected.
3. Crops have different production times, in other words, the time between planting and the last harvest varies.
4. Ecologically-based production constraints are modeled with the inclusion, in each rotation, of fallow and special crops for green manuring. Moreover, to reduce the occurrence of pests, crops belonging to the same botanic family can not be grown in sequence.
5. Available pieces of land can be heterogeneous due to climatic characteristics. For example, in a specific location, a subset of available crops can be grown and these crops have certain yields. This set of crops and their respective yields can be different in another location with different climatic and or soil characteristics.

In Santos et al. (2008), similar production constraints are considered (characteristics 2–4), but there are no demands associated with the crops, and both climate and yields for each available piece of land are the same. Indeed, in that case, the size of the plots were not decision variables but instead, were defined *a priori*. The goal was to maximize land occupation while respecting constraints 2–4 and additional adjacency constraints stating that adjacent plots cannot grow, simultaneously, crops belonging to the same botanical family. In this paper, there is not the notion of adjacency, since the plots are not known *a priori* but rather defined during the optimization process. This decision is made so that the yields are such that a known demand is met (characteristic 1 mentioned above).

The remainder of this paper is organized as follows. In the next section, we propose a linear mixed-integer formulation for the SVCSP. First, we detail the concept of crop rotation and propose linear constraints that model the set of feasible rotations which respect ecologically-based production conditions (characteristic 4 listed above) while considering that crops can be grown in specific periods of the year and that they have different growing times (characteristics 2 and 3). This crop rotation schedule model is used within a mathematical formulation for the complete sustainable vegetable crop demand supply problem. Extensions and additional comments concerning the model are presented in Section 3. The proposed formulation has an exponential number of variables and is, therefore, solved with a column-generation approach, which is presented in Section 4, as well as heuristic approaches to cope with the extensions discussed in the previous section. Section 5 presents computational results of the proposed methodology when applied to instances based on real-data obtained from an ecologically-based vegetable grower established in the countryside in Brazil. Section 6 ends this paper with some conclusions.

2. Mathematical formulation

In this section we propose a mathematical formulation for the SVCSP. The proposed model has an exponential number of variables, each one associated with a feasible crop rotation plan.

We first define linear constraints modeling the feasible crop rotations in the following subsection. Then, in Section 2.2, the complete model is presented and illustrated with a numerical example.

2.1. Sustainable crop rotation schedule

In an agricultural production system, a crop rotation schedule can be defined as the sequence of crops that should be planted, one after the other, in a given area. Many of the articles presented in Section 1 deal with grain crops. In this case, a crop rotation schedule is simply a list of crops in a specific sequence. For example, the rotation "corn–potatoes–potatoes" indicate that in year 1, corn should be planted. Then, in years 2 and 3, the land should be occupied with potatoes, returning to corn in year 4.

In the case of the SVCSP, we first define the time interval in which the sequence of crops will be repeated, T, which is divided into M equal time periods. A crop rotation schedule is a planting calendar for each of the crops cultivated in the rotation.

Ecologically-based production systems impose additional constraints on the crop rotations. In this study, among many possible sustainability practices, we concentrate on those mentioned in Section 1 which are presented in detail below:

(i) *Botanic family*: crops belonging to the same botanic family can not be planted in sequence.
(ii) *Green manure*: a number of crops for green manuring must be planted in each crop rotation. Moreover, green manures must be adequately spaced in time.
(iii) *Fallow*: in each crop rotation, a time of fallow must be respected. The required fallows might be subjected to specific durations and must be adequately spaced in time.

As an example, let us consider a crop rotation schedule of one year, divided into 12 periods of one month ($M = 12$). There are three crops, X, Y and Z with production times of 5, 4 and 2, respectively. Crops X and Y belong to the same botanic family while Z is a crop for green manuring. Crop X can be planted between January (month 1) and July (month 7), while crops Y and Z can be planted all year round. Finally, let us assume that one green manure and one month of fallow (F) are mandatory per schedule. Fig. 1 presents a feasible crop rotation schedule for this example:

In the crop rotation schedule presented in Fig. 1, crop X is planted in month 3 and occupies the land until period 7 (since its production time is 5 months). In period 8, crop Z is planted, staying in the land until period 9. In period 10, crop Y is planted and its final harvesting occurs in period 1 (of the next production year). In period 2, there is one month fallow (F). Note that each crop respects its planting times and crops X and Y (which belong to the same botanic family) are never planted in sequence. Moreover, as desired, in each crop rotation a green manure crop (Z) is cultivated and there is a period associated with fallow (F).

In the remainder of this section, we propose a linear model to describe crop rotations that respect the characteristics listed above. This is done with the help of the following decision variables:

$x_{ij} = 1$, if crop i is planted in period j, 0 otherwise, $i = 1 \ldots n$, $j \in I_i$,

where I_i is the set of periods in which the planting of crop i is allowed. Other parameters are detailed below:

M number of periods in each crop rotation (in a predefined unit of time);
C set of crops that can be selected for planting, not including the ones for green manuring;
G set of crops that can be selected for green manuring;
N cardinality of $C \cup G$;
$n = N + 1$ hypothetical crop associated with fallow;
NF number of botanic families;
$F(p)$ set of crops of botanic family p, $p = 1 \ldots NF$;
t_i production time of crop i, including the time estimated for preparing the soil and harvesting;

The model that describe the set of feasible crop rotations can then be written as

$$\sum_{i=1}^{n} \sum_{r=0}^{t_i-1} x_{i,j-r} \leqslant 1, \quad j = 1 \ldots M, \tag{1}$$

$$\sum_{i \in F(p)} \sum_{r=0}^{t_i} x_{i,j-r} \leqslant 1, \quad p = 1 \ldots NF, \; j = 1 \ldots m, \tag{2}$$

$$\sum_{i \in G} \sum_{j \in I_i} x_{ij} = 1, \tag{3}$$

$$\sum_{j=1}^{M} x_{nj} = 1, \tag{4}$$

$$x_{ij} \in \{0, 1\}, \quad i = 1 \ldots n, \; j \in I_i, \tag{5}$$

where in constraints (1) and (2), $j - r$ is replaced by $j - r + M$, whenever it is less or equal to zero.

Constraints (1) ensure that at most a crop is cultivated at each period while constraints (2) forbid crops of the same botanic family to be cultivated one immediately after the other. Constraints (3) and (4) force the planting of single green manure and a time of fallow at each crop rotation. The model above allows for solutions in which a given period has no associated crop. This is equivalent to allowing more times of fallow in each schedule, which poses no problem.

In model (1)–(5), the number of mandatory green manures and fallows has been set at one for each rotation period. Depending on the crop rotation length, these numbers might need to be increased. In this case, constraints (3) and (4) can then be written as:

$$\sum_{i \in G} \sum_{j \in I_i} x_{ij} = g, \tag{7}$$

$$\sum_{j=1}^{M} x_{nj} = f, \tag{8}$$

where g is the number of crops in G that must be planted per crop rotation, and f is the number of fallows per crop rotation schedule. In this case, one might need to ensure a minimum time delay between the planting of two green manures or two fallows. This can be done with the following additional constraints:

$$\sum_{i \in G} \sum_{r=0}^{tg} x_{i,j-r} \leqslant 1, \quad j = 1 \ldots M, \tag{9}$$

$$\sum_{r=0}^{tf} x_{n,j-r} \leqslant 1, \quad j = 1 \ldots M, \tag{10}$$

where tg and tf are the minimum time delays for green manures and fallows.

The proposed formulation can be easily adapted to the case where one is not concerned with the planning of a crop rotation but with the definition of a crop sequence schedule. Model (1)–(5) does not include demand constraints. In the next section, a model including them is developed. This new formulation contains

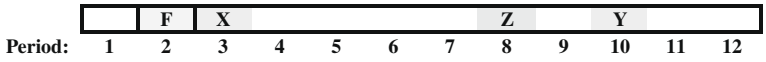


Fig. 1. A feasible one year crop rotation schedule.

6	**F**	**X**		*2*	*4*	*2*	**Z**		**Y**		
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

Fig. 2. Representation of a feasible one year crop production schedule.

an exponential number of variables and is, therefore, solved within a column-generation approach.

2.2. Crop demand supply problem

A crop rotation schedule is basically a planting calendar indicating when each crop should be planted. Since the planting of a crop in a given period implies one or more subsequent harvesting periods, with expected productivities, one might easily convert the crop rotation schedule (or planting calendar) into a *crop production schedule*, which is a harvesting calendar indicating the amount (in mass, volume, units or other appropriate measure) of each crop that is being produced in each period.

Consider again the example presented in the previous section and illustrated in Fig. 1. Assume that crop X has its first harvesting three months after planting and other harvesting occurs monthly, until the final harvesting, five months after the time of planting. The productions in months 3, 4 and 5 after planting are 1, 2 and 1 kilogram per square meter of planted area. Crop Y has only one harvest occurring in the last period of its production time of the four month and its production is estimated at 3 kilograms per square meter. With the use of this data, and assuming the planting calendar presented in Fig. 1 has been used in an area of 2 meter^2, we can express the planting and harvesting calendar (or production scheduling) in Fig. 2. The figures in italic represent the production per meter^2 of the harvesting crops.

As before, crop X is planted in month 3, crop Z in month 8, and crop Y in month 10. The new information in the figure concerns the yields of each crop in each period. Indeed, we can now observe that in months 5, 6 and 7, there is a production of 2, 4 and 2 kilograms of crop X, respectively. Analogously, in month 1 there is a production of 6 kilograms of crop Y. The production of crop Z is not indicated since it is being used just for green manuring and has no associated demand.

In order to model the SVCSP, one must match, at each period, the production schedule with the demands to be met of each crop. Let C be a set of crops with a known demand and $A_1,\ldots,A_L$ be the respective sizes of cropping areas (which can be further divided in the actual plots). Due, mainly, to possible heterogeneities in climate and soil conditions, the set of available crops and the associated productivity might differ for each cropping area. Let:

C_k, be the set of crops that can be planted in cropping area k;
S_k, the set of feasible production schedules associated with cropping area k;

Now, let a^s_{ijk} be the amount of crop $i \in C_k$ produced in period j per square meter of land in area k to which is assigned a crop rotation schedule s. Defining variables:

λ_{ks} size of the plot associated with crop schedule s in area k.

We can write the SVCSP as:

$$\max \quad \sum_{k=1}^{L}\sum_{s\in S_k} r_{ks}\lambda_{ks} \tag{11}$$

$$\text{s.t.} \quad \sum_{k=1}^{L}\sum_{s\in S_k} a^s_{ijk}\lambda_{ks} \geqslant d_{ij}, i \in C, \quad j = 1\ldots M, \tag{12}$$

$$\sum_{s\in S_k}\lambda_{ks} \leqslant A_k, \quad k = 1\ldots L, \tag{13}$$

$$\lambda_{ks} \geqslant 0, \quad s \in S_k, \quad k = 1\ldots L, \tag{14}$$

where d_{ij} is the demand of crop i in period j, given in appropriate units (kilogram, pack, etc.).

Parameter r_{ks} is the return obtained when production schedule $s \in S_k$ is used in one square meter of cropping area k. The goal of model (11)–(14) is to maximize this return which can be related, for example, to the total production volumes or revenues. Constraints (12) ensure that the demands for all crops, at all periods are satisfied while constraints (13) ensure that only the available land at each area is used.

As mentioned earlier, a production schedule is completely defined once a crop rotation schedule has been chosen. Indeed, consider:

o_i number of periods between the planting and the first harvesting of crop i.
p_{ikr} production per m^2 of the rth harvesting of crop i in the area k, for $r = 1\ldots t_i - o_i - 1$.

Given the crop rotation schedule $\mathbf{x}^s_k = (x^s_{ijk})$, we have:

$$a^s_{ijk} = p_{irk}\hat{x}^s_{i,j-o_i-r,k}, \quad i \in C_k,\ r = 1\ldots t_i - o_i - 1,\ j - o_i - r \in I_i,$$
$$s \in S_k,\ k = 1\ldots L.$$

For the case where one wishes to maximize the revenue obtained by the crop rotation schedules for the L areas, the income r_{ks} obtained for a production schedule $\mathbf{a}^s_k = a(x^s_{ijk})$ can be written as:

$$r_{ks} = \sum_{i\in C_k}\sum_{j\in I_i}\sum_{r=1}^{t_i-o_i-1} c_{ij}p_{irk}\hat{x}^s_{i,j-o_i-r,k}, \tag{15}$$

where c_{ij} is the unit price (per kilogram, per pack etc.) of crop i at period j.

Therefore, in terms of a crop rotation schedule $x^s_k = (x^s_{ijk})$, problem (11)–(14) becomes:

$$\max \quad \sum_{k=1}^{L}\sum_{s\in S_k}\sum_{i\in C_k}\sum_{j\in I_i}\sum_{r=1}^{t_i-o_i-1} c_{ij}p_{irk}\hat{x}^s_{i,j-o_i-r,k}\lambda_{ks} \tag{16}$$

$$\text{s.t.} \quad \sum_{k=1}^{L}\sum_{s\in S_k} p_{ir}\hat{x}^s_{i,j-o_i-r,k}\lambda_{ks} \geqslant d_{ij},$$
$$i \in C,\ j - o_i - r \in I_i,\ r = 1\ldots t_i - o_i - 1, \tag{17}$$

$$\sum_{s\in S_k}\lambda_{ks} \leqslant A_k, \quad k = 1\ldots L, \tag{18}$$

$$\lambda_{ks} \geqslant 0, \quad s \in S_k,\ k = 1\ldots L, \tag{19}$$

where $j - o_i - r$ is replaced by $j - o_i - r + M$, whenever it is less or equal to zero.

Example. Consider 3 cropping areas with sizes $A_1 = 6$, $A_2 = 4$ and $A_3 = 5$ m^2. There are 8 crops, which are listed in Table 1. For each crop i, the table presents the botanic family (F_i) to which it belongs, the set of possible planting dates (I_i), the production times (t_i), the number of periods before harvestings allowed (o_i) and the expected partial harvestings values, p_{ir}. For example, as $o_1 = 2$ and $t_1 - o_1 = 2$, crop 1 needs 2 periods between the planting period and its first harvest and has 2 periods of harvests.

Table 1 also presents the demands for each one of crops 1–6, at each period. Crops 5 and 6 are associated with green manuring and have no associated demand.

Table 1
Problem data: crop data and demands.

Crop	Crop data							Demand per period											
	F_i	I_i	t_i	o_i	p_{i1}	p_{i2}	p_{i3}	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1–7; 1–12	4	2	1	1				1	1								
2	1	1–9	3	2	1						1								
3	2	3–11	3	1	1	1		2			1	1			1	1			
4	2	2–11	4	1	1	2	1	4				8	6	8			2	8	10
5	3	7–12; 1–5	3	1	1	1				1	1								
6	3	3–10	3	1	2	1					2	2	2	1	1	2	2	2	2
7	4	2–12	3	2	1														
8	4	8–12; 1–6	2	1	1														

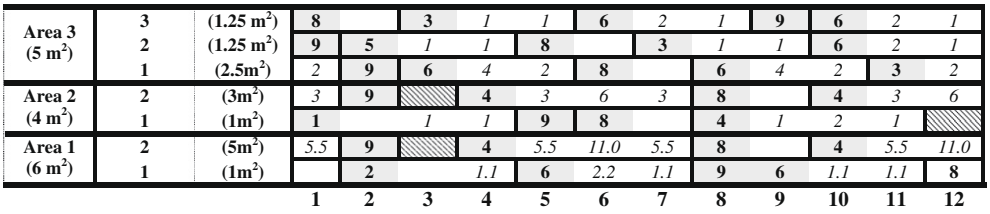


Fig. 3. Crop production schedules for areas A_1, A_2 and A_3.

Due to climatic and soil conditions, crop 1 cannot be planted in area 1. Analogously, crops 2, 3 and 5 are not compatible with area 2 while area 3 cannot grow crops 2 and 4. Moreover, the productivity in areas 1, 2 and 3 are 110%, 100%, and 80% of the basis values presented in Table 1, respectively. This way, the productivities of crop 1, for example, in area 1 are $p_{111} = 1.1$, $p_{121} = 1.1$, $p_{131} = 0$; in area 2, $p_{112} = 1$, $p_{122} = 1$, $p_{132} = 0$ and, in area 3, $p_{113} = 0.8$, $p_{123} = 0.8$ and $p_{133} = 0$.

Forcing one period of fallow (represented as crop 9) the solution presented in Fig. 3 is the one that maximizes the total non-weighted production ($c_{ij} = 1$, for all i and j). The figure uses the notation already introduced in Fig. 2 where the italic numbers correspond to the harvests. In the optimal solution, the sizes of cropping areas 1 and 2 have been divided into two plots with different crop rotation schedules while the area of area 3 has been divided into three plots. The values of variables in the optimal solution are: $\lambda_{11} = 1$, $\lambda_{12} = 5$, $\lambda_{21} = 1$, $\lambda_{22} = 3$, $\lambda_{31} = 2.5$, $\lambda_{32} = \lambda_{33} = 1.25$.

Clearly, model (11)–(14) contains a very large number of variables due to the fact that the number of possible crop schedules, at each area, grows exponentially with the considered number of crops and periods. For this reason, we propose a column-generation approach to solve this problem. The method is described in the Section 4.

3. Model discussion and extensions

In the previous section, a linear program has been presented for the SVCSP. Next, we further analyze the proposed model. We first note, in Section 3.1, that model (11)–(14) can be simplified in the case of homogeneous areas. Then, in Section 3.2, we discuss additional characteristics of the SVCSP found in practice and how they could be integrated in the model. Finally, Section 3.3 discusses a practical question concerning the number of production schedules generated.

3.1. Homogeneous areas

If there is no distinction among plots, i.e., if any crop can be planted in any plot yielding the same productivity, model (11)–(14) can be simplified and the plots can be treated as a single continuous arable area. Consider $A = \cup A_k$. Since a given crop production schedule now yields the same production schedule, regardless of the actual plot on which it is used, we can ignore index k in model (11)–(14), obtaining:

$$\max \quad \sum_{s\in S} c_s \lambda_s \tag{20}$$

$$\text{s.t.} \quad \sum_{k=1}^{L} \sum_{s\in S} a_{ij}^s \lambda_s \geqslant d_{ij}, \quad i \in C,\ j = 1 \ldots M, \tag{21}$$

$$\sum_{s\in S} \lambda_s \leqslant A, \tag{22}$$

$$\lambda_s \geqslant 0, \quad s \in S,\ k = 1 \ldots L. \tag{23}$$

The new variables λ_s indicate the amount of the available area in which crop schedule s is used. S is the set of all possible schedules. Let $\hat{\lambda}_1, \ldots, \hat{\lambda}_K$ be an optimal solution to model (20)–(23), the actual land division on each plot can be obtained with the help of the following linear system:

$$\sum_{k=1}^{L} \lambda_{ks} = \hat{\lambda}_s, \quad s = 1 \ldots K,$$

$$\sum_{\lambda_{ks}} \leqslant A_k, \quad k = 1 \ldots L,$$

$$\lambda_{ks} \geqslant 0, \quad s = 1 \ldots K,\ k = 1 \ldots L.$$

3.2. Unmet demand and upper bounds on demands

In practice, it might be interesting to allow solutions which do not meet all demand but, instead, make a better use of the productive land. This situation can be incorporated into model (11)–(14) with the inclusion of artificial variables in constraints (12). To limit the amount of non-supplied demand, these variables can be bounded or, instead, penalty terms can be associated with them in the objective functions. Considering this latter case, model (11)–(14) can be rewritten as:

$$\max \quad \sum_{k=1}^{L}\sum_{s\in S_k} r_{ks}\lambda_{ks} - \sum_{i\in C}\sum_{j=1}^{M} q_{ij}\theta_{ij} \tag{24}$$

$$\text{s.t.} \quad \sum_{k=1}^{L}\sum_{s\in S_k} a_{ij}^{s}\lambda_{s} \geqslant d_{ij} - \theta_{ij}, \quad i \in C,\ j = 1\ldots M, \tag{25}$$

$$\sum_{s\in S_k}\lambda_{ks} \leqslant A_k, \quad k = 1\ldots L, \tag{26}$$

$$\theta_{ij} \geqslant 0, \quad \lambda_{ks} \geqslant 0,\ s \in S_k, i \in C,\ j = 1\ldots M,\ k = 1\ldots L, \tag{27}$$

where θ_{ij}, is the amount of non-supplied demand of crop i in period j, and q_{ij} is the associated penalty.

Besides allowing part of the demand to be unmet, it might be convenient to establish an upper bound on the production. The motivation for this would be to limit the production of the crops to levels that can be absorbed by the farmers' clients. This can easily be done with the introduction of upper bounds on constraints (12). In Section 5, the effect of allowing unmet demands and upper bounds on production is studied.

3.3. Considerations on the number of plots

Model (11)–(14) imposes no constraints on the number of production schedules than can be used in a given area, allowing solutions with a very large number of plots. In order to reduce the occurrence of these solutions, which might be impractical in real-life situations, a penalty z_k can be associated with each variable $\lambda_{ks} > 0$. The original objective function is then replaced by:

$$\max \quad \sum_{k=1}^{L}\sum_{s\in S_k}(r_{ks}\lambda_{ks} + z_k\delta_{ks}) \tag{28}$$

with the following additional constraints:

$$\lambda_{ks} \leqslant A_k\delta_{ks}, \quad \delta_{ks} = \{0,1\},\ s \in S_k,\ k = 1\ldots L. \tag{29}$$

The penalty terms z_k are selected according to the desired trade-off between the number of plots in the solution and the obtained return. Formulation (28), subject to constraints (12), (13) and (29) is no longer a linear programming model but a more complex mixed-integer problem. In Section 4, we discuss heuristic strategies for its resolution.

4. Solution approaches

Model (11)–(14) can be seen as a master program in the column generation framework, since it is a linear optimization problem with a very large number of columns, which can be obtained by solving subproblems (Lübbecke and Desrosiers, 2005). The column-generation approach uses a *restricted* version of this master program, in which only a small subset of variables (or columns) is considered. New columns are iteratively added to the restricted master problem, while the procedure improves the current solution. When no improving columns can be found, the current restricted master solution is also the solution of the master problem.

In our case, each column in the master problem is associated to a crop rotation schedule for one of the areas k. As we only consider feasible crop rotations that respect ecologically-based criteria (i)–(iii), we can write the pricing problem as the problem of finding a crop rotation schedule that maximizes the corresponding reduced cost, respecting constraints (1)–(5), restricted to set C_k and crops for green manure for area k (G_k). By associating dual variables π_{ij} with constraints (12) and dual variables α_k with constraints (13), the column that gives the largest reduced cost for the cropping area k can be obtained by solving the following pricing problem P_k:

$$\max \quad \sum_{i\in C_k}\sum_{j\in I_i}\sum_{r=1}^{t_i - o_i - 1}(c_{i,j-r} - \pi_{i,j-r})p_{irk}x_{ijk} - \alpha_k$$

subject to that (x_{ijk}) satisfies (1)–(5) where only crops in $C_k \cup G_k$ are considered.

An algorithm for the used column-generation approach is presented below:

Column-generation algorithm

1. Choose a set of initial columns to the restricted master problem.
2. Solve the restricted master problem and obtain (π, α), the dual variables associated with constraints (12) and (13), respectively.
3. Solve the pricing problem P_k, for $k = 1\ldots L$, and obtain the reduced costs $\hat{c}_{ks}$ and the column $\mathbf{a}_k^s$.
4. If, for $k = 1\ldots L$, $\hat{c}_{ks}$=0, then **stop**. The current solution is optimal. Otherwise, insert column $\mathbf{a}_k^s$ with relative costs $\hat{c}_{ks} > 0$ in the restricted master problem and go to *step 2.*

At the end of the column-generation algorithm, we have a set of feasible production schedules ($\mathbf{a}_k^s$) associated with given plot sizes ($\hat{\lambda}_{ks}$). There might be a large number of $\hat{\lambda}_{ks} > 0$, a significant number of which having tiny values. Next, we propose some strategies to deal with this problem.

Given ($\mathbf{a}_k^s, \hat{\lambda}_{ks}$), a solution for (11)–(14) containing a large number of values $\hat{\lambda}_{ks} > 0$, a simple way to reduce the number plots is to set each $\hat{\lambda}_{ks} < \delta$ at zero, where δ is a minimum desired plot size. We name this simple strategy *δ-plot elimination.* Another heuristic, named *plot-minimization*, is to select, among the columns (production schedules) present in the optimal restricted master problem, the minimum number of plots which is enough to meet the demand. This can be done with the aid of the following mixed-integer problem:

$$\min \quad \sum_{k=1}^{L}\sum_{s=1}^{\hat{S}_k}\delta_{ks} \tag{30}$$

$$\text{s.t.} \quad \sum_{k=1}^{L}\sum_{s\in S_k} a_{ijk}^{s}\lambda_{ks} \geqslant d_{ij}, \quad i \in C,\ j = 1\ldots M, \tag{31}$$

$$\sum_{s\in \hat{S}_k}\lambda_{ks} \leqslant A_k, \quad k = 1\ldots L, \tag{32}$$

$$\lambda_{ks} \leqslant A_k\delta_{ks}, \quad k = 1\ldots L, \tag{33}$$

$$\delta_{ks} = \{0,1\}, \quad \lambda_{ks} \geqslant 0,\ s \in \hat{S}_k,\ k = 1\ldots L. \tag{34}$$

Both *δ-plot elimination* and *plot-minimization* procedures lead to an increase in unused area and unmet demand. General *residual strategies* for obtaining a good use of the unused area (and possibly reducing the unmet demand) can be developed. For example, let $\hat{\lambda}_{ks}$, $s = 1\ldots S_k$, $k = 1\ldots L$, be an optimal solution of the master problem (11)–(14) and define the residual of crop i at period j as:

$$rd_{ij} = d_{ij} - \sum_{k=1}^{L}\sum_{s\in\hat{S}_k} a_{ijk}^{s}\hat{\lambda}_{ks}.$$

The resulting unused area can be used to increase some plot sizes in order to reduce the unmet demands. Priority can be given to crops with higher residual demands. Another option is to solve a modified version of model (11)–(14) in which the demand values are changed for their residual counterparts, rd_{ij}. This procedure can be used recursively, until there is no area left to be assigned or another appropriate stopping criterion is reached.

Naturally, other procedures for dealing with the problem mentioned in Section 3.3 exist. Among them, one can include alterna-

Table 2
Results for instances with $L = 1$.

$L = 1$					
Crop	Demand	*time*	*# plots*	*%resid*	*%oarea*
12	*d200*	250.20	224.30	0.73	72.85
	$d\infty$	757.25	119.80	0.00	100.00
16	*d200*	910.35	395.40	0.47	86.88
	$d\infty$	2331.47	237.00	0.00	100.00
20	*d200*	3283.76	494.70	0.50	100.00
	$d\infty$	3075.58	426.30	0.00	100.00
24	*d200*	3230.28	549.40	0.60	100.00
	$d\infty$	3601.36	528.22	0.00	100.00

Table 3
Results for instances with $L = 3$.

$L = 3$					
Crop	Demand	*time*	*# plots*	*%resid*	*%oarea*
12	*d200*	257.37	266.10	0.51	84.53
	$d\infty$	646.62	116.89	0.00	100.00
16	*d200*	811.76	335.40	0.63	91.82
	$d\infty$	1283.94	244.50	0.00	100.00
20	*d200*	2630.80	497.90	0.49	100.00
	$d\infty$	3430.26	457.56	0.00	100.00
24	*d200*	3230.28	549.40	0.60	100.00
	$d\infty$	3596.96	594.50	0.00	100.00

Table 4
Results for instances with $L = 5$.

$L = 5$					
Crop	Demand	*time*	*# plots*	*%resid*	*%oarea*
12	*d200*	129.83	249.60	0.68	64.81
	$d\infty$	708.97	135.60	0.00	100.00
16	*d200*	766.70	394.50	0.48	94.30
	$d\infty$	2722.03	241.50	0.00	100.00
20	*d200*	2062.26	502.40	0.67	100.00
	$d\infty$	2947.01	424.80	0.00	100.00
24	*d200*	3168.13	546.67	0.60	100.00
	$d\infty$	3612.39	635.89	0.00	100.00

tive heuristics or even the exact resolution of the proposed mixed-integer formulation with the aid of a branch-and-price algorithm, obtained by adapting the proposed column-generation within a branch-and-bound technique.

In Section 5, we present results for both *δ-plot elimination* and *plot-minimization* heuristics. The results are presented in terms of a worst-case analysis, where residual heuristics are not used.

5. Computational experiments

We ran several computational experiments in order to verify the performance of the column-generation approach proposed in Section 3, as well as the efficacy of the *δ-plot elimination* and *plot-minimization* heuristics presented in the same section. All methods were implemented in C++ and linear and mixed-integer models were solved with the aid of CPLEX 11.0 (ILOG, 2007). The tests were run on an Intel Core Duo machine, with 2.33 GHz clock and 4 Gb of RAM.

A planning horizon of two years, divided into weeks, was used. A single green manure and one month's fallow were adopted at each rotation schedule. Four groups of instances were tested, with a varying number of crops ($N = 12$, 16, 20 and 24). Crop data, listed in Tables A.1 and A.2 in the Appendix A, as well as production parameters (planning horizon, time discretization, number of green manures and fallows) were obtained in an ecologically-based production unit situated in the city of Barbacena (21° 13′ 30″S and 43° 46′ 25″ W, 1165 meters above sea level), in the countryside of Brazil. For each group, we tested the performance of the algorithm for a different number of cropping areas: $L = 1$, 3 or 5. Case $L = 1$ is associated with the homogeneous area problem, detailed in Section 3.1. Cases with $L = 3$ and $L = 5$ corresponding to heterogeneous areas were manually generated. A set of crops were selected as feasible for each cropping area, with varying productivity (with a maximum variation of 30% regarding the data in Table A.2). The demand was also generated randomly according to the possible planting and harvesting periods of each crop. Finally, for each case, we tested the performance of the methods both when no upper bounds on production existed and when the production was limited to 200% of the demand, for each crop, at each period. These cases are presented in the tables as $d\infty$ e *d200*, respectively.

The combination of the different parameters on the number of crops ($N = 12$, 16, 20 and 24), number of cropping areas ($L = 1$, 3 and 5) and upper bounds on productions ($d\infty$,*d200*) gives rise to 24 different classes, each one containing 50 instances. In all cases, we have included artificial variables to allow for unmet demands (see Section 3.2), assuming equal penalties for the unmet demands of the different crops.

Tables 2–4 present the results for the column-generation algorithm. For each class of instances, the tables list the execution times (*time*), the average number of plots in the optimal solution (*# plots*), the average percentage of unmet demand (*%resid*), and the percentage of the total area that is used (*%oarea*). In these tests, the computational times were limited to 1 hour.

As shown in the tables, the algorithm was able to obtain solutions within the allowed time limit for most of the instances. The running times increase with the number of crops but showed no clear dependency on the number of cropping areas. In general, classes with an upper bound on the production are solved faster. In cases labeled d∞, the constraints on available arable land are

Table 5
Results for the *α-plot elimination* with $\delta = 1$ meter^2.

Crop	*dmax*	$L = 1$			$L = 3$			$L = 5$		
		# plots	*%darea*	*%lostde*	*# plots*	*%darea*	*%lostde*	*# plots*	*%darea*	*%lostde*
12	*d200*	201.40	0.11	0.86	233.10	0.15	1.18	224.80	0.12	1.54
	$d\infty$	114.70	0.03	0.15	110.00	0.03	0.50	128.30	0.04	0.38
16	*d200*	336.60	0.26	0.68	303.90	0.14	1.19	351.10	0.19	0.62
	$d\infty$	221.50	0.07	0.42	232.00	0.06	0.42	230.10	0.06	0.38
20	*d200*	432.00	0.30	1.30	442.30	0.25	0.62	443.50	0.27	1.06
	$d\infty$	378.80	0.23	1.22	420.33	0.18	0.93	390.60	0.16	0.70
24	*d200*	478.20	0.33	1.32	478.20	0.33	1.32	487.89	0.28	1.40
	$d\infty$	469.44	0.26	0.93	529.80	0.31	1.49	561.67	0.35	1.68

Table 6
Results for the *plot-minimization*.

Crop	*dmax*	$L = 1$			$L = 3$			$L = 5$		
		time	*# plots*	*%oarea*	*time*	*# plots*	*%oarea*	*time*	*# plots*	*%oarea*
12	*d200*	259.99	73.70	38.03	591.09	86.70	46.97	333.66	75.40	35.65
	$d\infty$	409.33	33.80	83.06	1026.59	36.22	81.89	1442.74	37.00	69.48
16	*d200*	463.94	137.00	53.56	844.12	120.70	52.07	764.38	138.50	58.76
	$d\infty$	1340.40	55.30	80.84	1800.03	61.40	88.63	1800.06	69.90	95.23
20	*d200*	1551.62	181.20	73.48	1323.63	192.40	82.38	1638.11	190.90	79.36
	$d\infty$	1800.09	169.60	100.00	1800.10	195.33	95.34	1800.22	189.50	90.63
24	*d200*	1540.84	247.80	89.49	1540.84	247.80	89.49	1800.18	301.78	97.50
	$d\infty$	1800.41	278.00	100.00	1800.56	272.90	100.00	1800.46	302.89	91.11

active (the area is completely used) while when the production is limited to 200%, there is an amount of unused area for cases with fewer crops.

The number of cropping areas has no clear effect on the number of plots in the optimal solution, but these are strongly dependent on the number of crops. Indeed, the larger the number of crops be, the larger the number of different crop rotation schedules used, which can reach, on average, 600 schedules for the larger instances ($N = 24$). In order to cope with this problem, the procedures presented in Section 4 were used.

Applying the *δ-plot elimination* with $\delta = 1$ meter^2, yields the results in Table 5, where for each class the resulting number of plots (*# plots*) are presented, the area associated with discarded plots (*%darea*), in percentage terms of the total area, and the percentage of unmet demand due to these plots (*%lostde*).

As can be seen in Table 5, the sole application of the simple *δ-plot elimination* strategy was able to reduce the number of plots by 10%. However, since the areas associated with the discarded plots are small, the net effect on the demand supply is not very significant (around 1%, in average).

Table 6 shows some results for the *plot-minimization* heuristic. The table presents the heuristic solving times (limited to 0.5 hour), the resulting number of plots (*# plots*), and the final percentage of occupied area (*%oarea*).

The reduction in the number of plots now reaches 60% on average, in comparison to the initial solution. Since we force the demand supply in this heuristic, it might occur that there are still plots associated with tiny areas. The *δ-plot elimination*could, therefore, be applied to the obtained solution.

6. Conclusions

We have considered the sustainable vegetable crop demand supply problem (SVCSP), in which one must meet known crop demands while respecting ecologically-based production criteria. The problem has been modeled as a linear program with a very large number of variables, each one indicating the plot size associated with a feasible rotation schedule. A linear integer pricing problem to obtain these feasible schedules was proposed, which allowed the solving of the SVCSP by means of a column-generation approach. Post-optimization methodologies were discussed in order to improve the applicability of the obtained solutions. Computational tests on instances adapted from real-data of up to 24 crops indicated the efficacy of the proposed methodologies.

Table A.1
Data for the 26 crops: name, botanic family, appropriate planting periods and production time.

Crop		Botanic family		Planting period		Production time (weeks)
				Beginning	End	
1	Crisp head lettuce	1	Asteraceae	All year round		7
2	Loose leaf lettuce	1	Asteraceae	All year round		7
3	Butter head lettuce	1	Asteraceae	All year round		7
4	Endive	1	Asteraceae	All year round		9
5	Watercress	2	*Brassicaceae*	February	July	32
6	Chinese cabbage	2	*Brassicaceae*	February	September	32
7	Broccoli	2	*Brassicaceae*	February	October	20
8	Cauliflower	2	*Brassicaceae*	March	October	18
9	Beet	3	*Chenopodiaceae*	February	September	11
10	Spinach	3	*Chenopodiaceae*	February	September	20
11	Zucchini	4	*Cucurbitaceae*	October	February	14
12	Pumpkin	4	*Cucurbitaceae*	November	January	19
13	Cucumber	4	*Cucurbitaceae*	September	March	13
14	Garlic	5	*Liliaceae*	March	April	24
15	Onion	5	*Liliaceae*	March	July	24
16	Leek	5	*Liliaceae*	April	April	12
17	Okra	6	*Malvaceae*	November	January	27
18	Tomato	7	*Solanaceae*	All year round		24
19	Carrot	8	*Umbelliferae*	All year round		16
20	Parsley	8	*Umbelliferae*	October	February	21
21	Bean	9	*Leguminosae*	October	February	12
22	Black velvet bean	9	*Leguminosae*	October	January	16
23	Jack bean	9	*Leguminosae*	October	February	12
24	Lupine	9	*Leguminosae*	March	July	18
25	Hairy vetch	9	*Leguminosae*	March	July	20
26	Sunnhemp	9	*Leguminosae*	October	January	16

Table A.2
Data for the 21 crops with associated demand: name, measuring unit, number of periods between planting and first partial harvesting (o_i) and partial expected harvestings per square meter of planted area.

Crop		Measuring unit	Harvesting																			
			o_i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	Crisp head lettuce	Unities	5	9	3																		
2	Loose leaf lettuce	Unities	5	9	3																		
3	Butter head lettuce	Unities	5	9	3																		
4	Chicory	Unities	5	9	3																		
5	Watercress	Pack	12	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1
6	Chinese cabbage	Pack	12	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1
7	Broccoli	Pack	10	1	1	2	2	3	3	3	2	2	1										
8	Cauliflower	Unities	16	1	3																		
9	Beet	kilogram	8	1	2	1																	
10	Spinach	Pack	5	3	4	4	4	4	4	4	4	4	4	4	4	3	3	2					
11	Zucchini	kilogram	9	0.2	0.3	0.3	0.3	0.2															
12	Pumpkin	kilogram	16	0.5	0.5	0.5																	
13	Cucumber	kilogram	8	1	2	2	1	1															
14	Garlic	kilogram	23	0.3																			
15	Onion	kilogram	23	3																			
16	Leek	kilogram	8	3	3	3	3																
17	Okra	kilogram	14	0.2	0.3	0.3	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.2	0.2	0.2							
18	Tomato	kilogram	15	0.8	0.8	1	1	1.2	1.2	1	0.8	0.2											
19	Carrot	kilogram	13	1.5	2	1.5																	
20	Parsley	Pack	8	14	16	16	16	16	16	16	16	16	16	16	14	10							
21	Bean	kilogram	11	0.3																			

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Appendix A

For the computational experiments, we selected 26 crops which are cultivated at an organic farming unit in Barbacena, Brazil. Crops 22–26 are used for green manuring. The discretization time is one week. Data listed in Tables A.1 and A.2 are approximations of the real values. The production time for each crop includes the estimated planting and harvesting periods.

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