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A hybrid path-relinking method for solving two-stage stochastic integer problems

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Abstract

Path relinking has been used for solving deterministic problems by exploring the neighborhood of elite solutions in an intelligent way. We present an algorithm that combines a mixed-integer linear solver with a truncated path-relinking method in order to solve two-stage stochastic integer problems with complete recourse and first-stage integer variables. This method takes advantage of a possible scenario-based decomposition in an innovative way. Therefore, path relinking is used to combine optimized solutions from different scenarios in order to pursue good stochastic solutions. To assess the computational performance of this method, we use the stochastic lot sizing and scheduling problem dealing with perishable products. In this problem, first-stage decision variables are linked to production sequences and production quantities. After the uncertain demand is unveiled, the second-stage variables decide on the inventory usage. Computational results show a clear advantage of the proposed method when compared to a state-of-the-art mixed-integer linear solver.

Keywords: path relinking; mixed-integer solver; stochastic programming; lot sizing and scheduling; demand uncertainty

1. Introduction

Stochastic mixed-integer linear models have been used to formulate numerous planning problems, and a multitude of solution approaches are available to solve them. In the review of Bianchi et al. (2008), an exhaustive analysis of exact methods and metaheuristics that are used to solve such problems has been performed. Basically, almost every traditional metaheuristic and exact method has been adapted to solve these hard problems, such as tabu search (Costa and Silver, 1998), ant colony optimization (Gutjahr, 2003), and branch and bound (Gutjahr et al., 2000). Among these solution methods, some take advantage of the scenario sampling structure that characterizes a large portion of these problems. A paradigmatic example of an algorithm that uses this decomposition

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is the progressive hedging, which was first proposed by Rockafellar and Wets (1991). This method considers a set of representative scenarios, which should grasp the stochasticity in the second-stage parameters. Each of these scenarios is solved by means of a deterministic model that captures the related subproblem. By the end of the first step, a pool of solutions independently optimizing the scenarios is obtained. Afterwards, through an averaging procedure of all considered solutions, a compromise is obtained between all possible uncertain outcomes. Several improvements and enhancements of the base algorithm have been suggested (Lokketangen and Woodruff, 1996), and the readers are referred to Watson and Woodruff (2010) for an updated overview of this method.

This paper presents a hybrid solution method that also uses a scenario-based decomposition. However, this method takes advantage of the resulting subproblems by combining, through path relinking, the individual solutions of each scenario. This algorithm combines a truncated path-relinking method and mixed-integer programming-based method. It uses the deterministic integer variables values obtained from the solution of each stochastic scenario in order to approach the global optimum of the stochastic model. The truncated path-relinking is similar to a standard path-relinking, but stops the search in between the starting and guiding solutions. The proposed solution method is designed to solve two-stage stochastic integer problems with complete recourse and first-stage integer variables, which can be generally formulated as

$$\min(c \cdot x + b \cdot y) + \sum_{v \in [V]} \phi^v (f_v \cdot z_v) \tag{1}$$

subject to

$$(x, y, z_v) \in \mathfrak{D}_\upsilon \quad \forall v \in [V] \tag{2}$$

$$x,\ z_v \geq 0;\ y \in \{0, 1\}, \tag{3}$$

where $\mathfrak{D}_\upsilon$ is the set of feasible solutions for the decision variables (x, y, z_v). $[V]$ is the set of scenarios v, and c, b, f_v are scalars. Note that f_v is a stochastic parameter.

The first contribution of this paper is to extend a solution method (path relinking) that has been mainly used to solve deterministic problems to address stochastic ones. The second contribution of this paper relates to the hybrid nature of the extension. Therefore, this paper shows another possibility of combining exact and heuristic methods to solve stochastic problems.

The paper is organized as follows. Section 2 describes the proposed solution method. In Section 3, a mathematical description of the problem used to test the algorithm is given. In Section 4, the computational results are reported. Finally, Section 5 resumes the main findings and indicates paths for future research.

2. The hybrid path-relinking method

Path relinking is a method that explores the neighborhood of elite solutions in a systematic way. This method was proposed by Glover and Laguna (1993) and readers are referred to Resende et al.

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(2010) for a deeper understanding of this method and its variants, such as forward, backward, and mixed path-relinking. The key reasoning behind path relinking is that good solutions should share a similar structure among them. Thus, in the basic version of path relinking, two solutions are chosen at each time and the solution elements of one of these solutions are changed through a path that finalizes when the starting solution changes to the target solution. The usage of path relinking has been mostly associated with GRASP (Festa and Resende, 2009, 2011, 2013), but there are also other hybridizations in the literature focusing on more specific problems (Zeng et al., 2013).

We cross-fertilized this general path-relinking concept with the idea proposed in Rockafellar and Wets (1991): "The idea is that by studying the different sub-problems [scenarios] and their [sub]optimal solutions one may be able to discover similarities and trends and eventually come up with a 'well hedged' solution to the underlying problem, something which can be expected to perform rather well under all scenarios, relative to some weighting of scenarios." Therefore, in this paper, we reframe the idea of path relinking by applying it to stochastic mathematical problems represented by a set of scenarios with different probabilities.

The natural structure of the two-stage stochastic integer problems with continuous second-stage decision variables is used to develop a hybrid heuristic approach. As mixed-integer linear solvers do not take advantage of the structure of this kind of problems, we tackle this issue heuristically using a hybridization of path relinking with a solver that independently explores the set of scenarios. Our reasoning is that an optimal solution should potentially lie somewhere in between different scenario solutions. The first step of the algorithm is to optimize individually the deterministic problems associated with each scenario. Afterwards, solutions are ranked based on the objective function value of the stochastic problem in which the first-stage integer decisions are fixed to the values found for the respective scenario (stochastic evaluation). This ranking will define the sequence of guiding solution entering the path-relinking phase. Through path relinking, the integer part of the most promising solutions is combined and, again, the intermediate solutions are evaluated by the stochastic objective function with the deterministic equivalent model (DEM). The best solution found in all iterations through this last evaluation step is returned as the best. Figure 1 shows the main outline of the method. In this scheme, the first phase is performed with a branch-and-bound procedure. After exploring each scenario independently, a linear programming (LP) solver is used to evaluate solutions. The figure exemplifies the first iteration of the path relinking. After finding the best solution of this iteration, it will serve as a starting solution for the next one.

Note that all solutions found in the first step are ranked according to their performance in the stochastic setting and those with better performance are preferred. Nevertheless, in case the stopping criterion is not met, all solutions might end up being used in the path-relinking phase.

Throughout the algorithm execution, two different types of calls to the mixed-integer solver are executed. (1) A given scenario is solved with a certain demand realization and (2) the linear problem arising is solved when fixing the first-stage integer variables of the stochastic model (stochastic evaluation). These last subproblems are solved rather quickly in comparison to a complete scenario (solved in the first call).

Algorithm 1 describes the pseudocode of the hybrid path-relinking where these calls are used. In the first loop, a feasible solution is found for every demand scenario ($\bar{y}_v$) within a certain time limit

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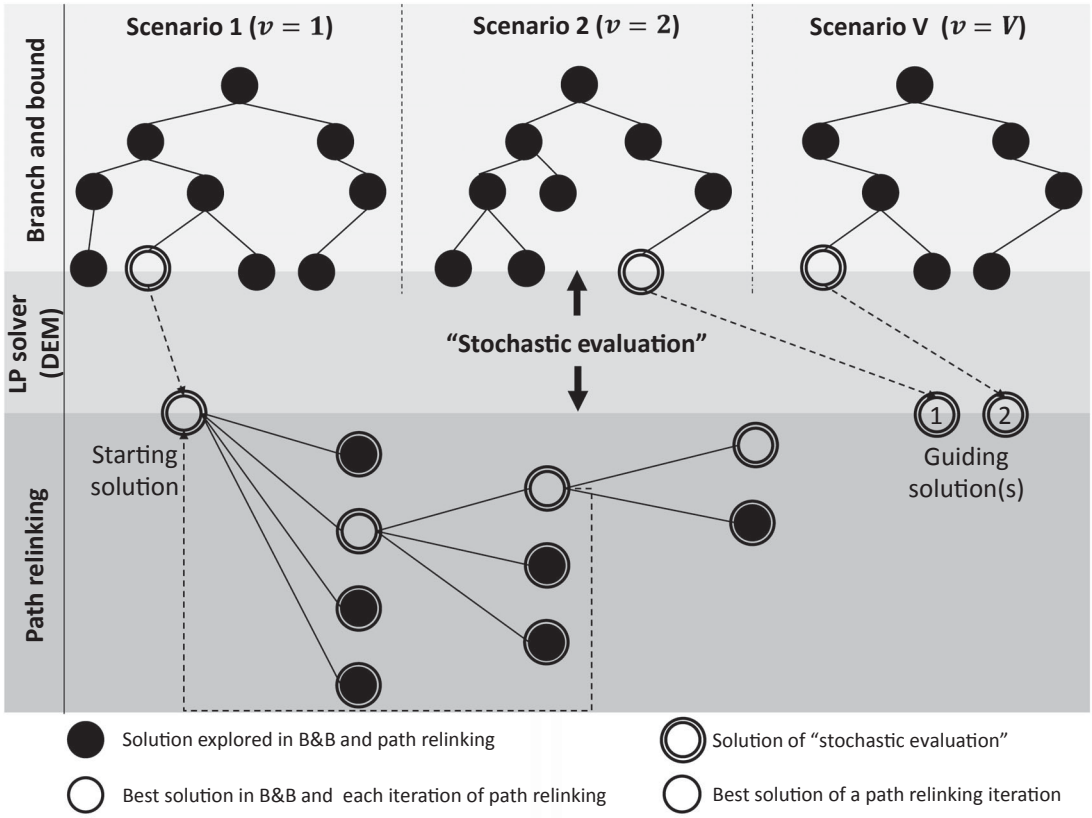


Fig. 1. Outline of the hybrid path relinking.

(branch-and-bound procedure of Fig. 1). These solutions are evaluated for the stochastic setting with the DEM having the first-stage integer variables fixed to the values found before (depicted with the LP solver step in Fig. 1). With this information, a truncated path-relinking is restarted multiple times with the most promising guiding solutions (using a "stochastic evaluation") until one of the two stopping criteria is met: there are no more solutions of scenarios to be served as guiding solutions or the last guiding solution did not lead to a better solution. The first starting (*str*) and guiding (*gdg*) solutions are set to the two best solutions found (according to the "stochastic evaluation"). New guiding solutions are inserted at each path-relinking restart and the starting solution is updated to the best solution found in each iteration. Within each path-relinking iteration, the vectors of first-stage integer decision variables from the starting and guiding solutions are compared. The set of positions with different values (δ) is used to transform the starting into the guiding solution until reaching a degree of resemblance set by the ϵ criterion. Setting ϵ to $|\delta|$ converts this truncated path-relinking into a standard one. All these steps are condensed into the path-relinking phase, as shown in Fig. 1. Every intermediate solution is evaluated for the stochastic setting. Note that the position i of the vector of first-stage integer decision variables $\vec{y}_v$ is denoted by $y_{i,v}$.

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Algorithm 1. Pseudocode for the hybrid path-relinking

for $v \in [V]$ **do**
  $\bar{y}_v := \arg\min_y(c \cdot x + b \cdot y + f_v \cdot z_v) : (x, y, z_v) \in \mathfrak{D}_\mathfrak{v}$;
$str := \arg\min_{v\in[V]}(c \cdot x + b \cdot \bar{y}_v + f_v \cdot z_v) : (x, z_v) \in \mathfrak{D}_\mathfrak{v}$;
$strB := str$;
$[V] := [V] \setminus \{str\}$;
$ObjG := +\infty$;
while $ObjG < ObjB$ *and* $[V] \neq \{\}$ **do**
  $gdg := \arg\min_{v\in[V]}(c \cdot x + b \cdot \bar{y}_v + f_v \cdot z_v) : (x, z_v) \in \mathfrak{D}_\mathfrak{v}$;
  $[V] := [V] \setminus \{gdg\}$;
  $str := strB$;
  $\delta := \{i = 1, ..., |\bar{y}_{str}| : y^i_{str} \neq y^i_{gdg}\}$;
  $ObjM, ObjB := +\infty$;
  while $|\delta| > \epsilon$ **do**
    for $i \in \delta$ **do**
      $\bar{y}_{str} := \{..., y^{i-1}_{str}, y^i_{gdg}, y^{i+1}_{str}, ...\}$;
      $Obj := \min(c \cdot x + b \cdot \bar{y}_{str} + f_v \cdot z_v) : (x, z_v) \in \mathfrak{D}_\mathfrak{v}$;
      if $Obj < ObjM$ **then**
        $ObjM := Obj$;
        $strM := str$;
        $i_{max} := i$;
    $\delta := \delta \setminus \{i_{max}\}$;
    if $ObjM < ObjB$ **then**
      $ObjB := ObjM$;
      $strB := strM$;
  if $ObjB < ObjG$ **then**
    $ObjG := ObjB$;
Output: $ObjG$

The best stochastic solution found within the multiple path-relinkings is returned as the solution to the problem ($ObjG$).

3. Stochastic lot sizing and scheduling problem dealing with perishable products

Within supply chain planning tasks, the lot sizing and scheduling problem is responsible for determining the size of each production lot and the sequence in which these lots are produced in a medium- to short-term planning horizon. Stochastic lot sizing and scheduling problems appear when parameters' uncertainty is taken into account. Lot sizing and scheduling problems are known to be NP-hard and therefore optimal solutions are difficult to obtain and prove even for medium-sized instances. Moreover, the stochastic version of this deterministic problem, where

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demand is unknown in advance, usually requires even more computational effort to be solved. We focus on the stochastic lot sizing and scheduling problem dealing with perishable products (S-LSP-PP) arising in the food consumer goods industries that use "direct store delivery." The main impact of this delivery practice is an increased responsibility of producers for all downstream processes of the supply chain until reaching the final customer. This responsibility entails a heavier burden in the case of food companies producing perishable products that have a limited shelf life, inhibiting the abusive use of intermediate storage to hedge against demand uncertainty. This lot sizing and scheduling problem was first introduced in Amorim et al. (2013). As this was the first time that this problem was discussed, no solution method was ever proposed to solve it.

The S-LSP-PP can be classified as a two-stage stochastic integer problem with complete recourse and first-stage integer variables. In this problem structure, first-stage (lot sizing and scheduling) decisions are responsible for the production planning without knowing the exact demand (tactical level), whereas second-stage variables allocate the inventory and production output to the realized demand (operational level).

3.1. Formal problem description

Consider products i, $j = 1, \ldots, N$ that are to be scheduled on $l = 1, \ldots, L$ parallel production lines over a finite planning horizon consisting of macroperiods $t = 1, \ldots, T$ with a given length. The changeover time and cost between products on a line depends on the sequence.

A macroperiod is divided into a fixed number of nonoverlapping microperiods with a variable length. Since the production lines can be independently scheduled, this division is done for each line separately. Let $[S_{lt}]$ denote the set of microperiods $s = 1, \ldots, S_{lt}$ belonging to macroperiod t and production line l. The number of microperiods of each macroperiod defines the upper bound on the number of products to be produced on each line. The length of a microperiod is linked to the decision variable accounting for the quantity of products produced. A product lot may continue over several micro- and macroperiods since setup carry-over is considered. Thus, a lot is independent of the discrete time structure of the macroperiods. Note that the lot sizing and scheduling time structure is based on the general lot sizing and scheduling structure for parallel lines (Meyr, 2002).

The demand for each product j at its fresher state in macroperiod t (d_{jt}^{0v}) is stochastic and obtained through the sampling of discrete scenarios $v = 1, \ldots, V$. Each of these scenarios has an associated probability ϕ^v, such that $\phi^v > 0, \forall v$ and $\sum_v \phi^v = 1$. In Amorim et al. (2013), the authors study the influence of different consumer purchasing behavior. For testing the hybrid path-relinking, several simplifications were considered. Therefore, no product families arrangements are considered, every product is assumed to have a fixed shelf life (u_j), consumer purchasing behavior related to picking up the fresher product available is disregarded, and the demand over the age of the product varies in the same manner as the price. The present study considers a linear willingness to pay shape for customers who are rather sensitive to product freshness. Therefore, we acknowledge the decreasing customer's value to an aging product. With Equation (4), the demand parameter for product j with age a in period t according to scenario v (given by d_{jt}^{av}) is calculated. Moreover, Fig. 2 presents an example of a demand curve plotted using the following equation:

$$d_{jt}^{av} = d_{jt}^{0v} - \frac{0.5 d_{jt}^{0v} a}{u_j - 1}. \tag{4}$$

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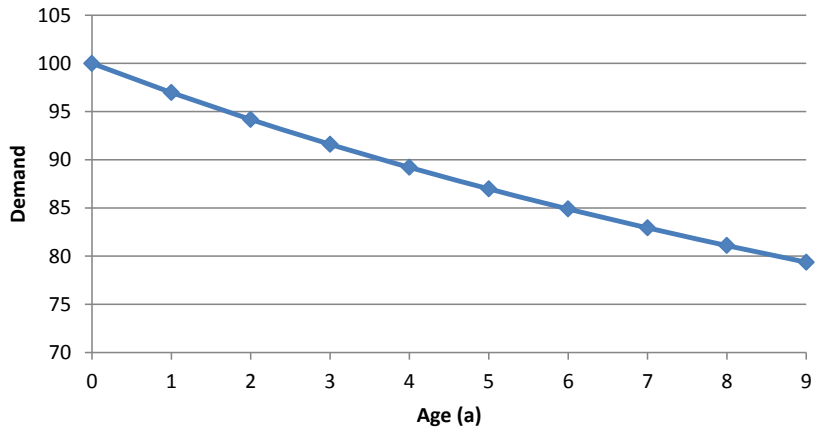


Fig. 2. Example of an age-dependent demand curve.

Consider the following indices, parameters, and decision variables.

Indices

l	parallel production lines
i, j	products
t	macroperiods
s	microperiods
a	age (in macroperiods)
v	scenarios

Parameters

$[S_{lt}]$	set of microperiods s within macroperiod t and line l
C_{lt}	capacity (time) of production line l available in macroperiod t
a_{lj}	capacity consumption (time) needed to produce one unit of product j on line l
c_{lj}	production costs of product j (per unit) on line l
p_j	price of each product j sold
$\bar{p}_j$	cost incurred for each product j spoiled
u_j	shelf-life duration of product j right after being produced (time)
m_{lj}	minimum lot size (units) of product j when produced on line l
$s_{lij}(\tau_{lij})$	sequence-dependent setup cost (time) of a changeover from product i to product j on line l
y_{lj0}	equals 1, if line l is set up for product j at the beginning of the planning horizon (0 otherwise)

First-stage decision variables

q_{ljs}	quantity of product j produced in microperiod s on line l
y_{ljs}	equals 1, if line l is set up for product j in microperiod s (0 otherwise)
z_{lijs}	equals 1, if a changeover from product i to product j takes place on line l at the beginning of microperiod s (0 otherwise)

Second-stage decision variables

w_{jt}^{av}	inventory of product j with age a available at macroperiod t in scenario v
ψ_{jt}^{av}	quantity of product j with age a delivered at macroperiod t in scenario v

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We denote a given set $\{1, 2, \ldots, M\}$ as $[M]$. Further note that variables w_{jt}^{av} and ψ_{jt}^{av} are only instantiated for a certain domain to ensure that no perished product is kept in stock or used to fulfill demand. Hence, the dynamic set $[A_w] = \{a \in \mathbb{Z}^+ | a \leq \min\{u_j, t-1\}\}$ and the set $[A_\psi] = \{a \in \mathbb{Z}^+ | a \leq \min\{u_j - 1, t-1\}\}$ are used depending on the corresponding decision variable. Without loss of generality, we assume that both inventory at the beginning and end of the planning horizon are null.

S-LSP-PP

$$\max \sum_v \phi^v \left(\sum_{j,t,a} p_j \psi_{jt}^{av} - \sum_{j,t} \bar{p}_j w_{jt}^{u_j,v} \right) - \sum_{l,i,j,s} s_{lij} z_{lijs} - \sum_{l,j,s} c_{lj} q_{ljs} \tag{5}$$

subject to

$$\sum_a \psi_{jt}^{av} \leq d_{jt}^{0v} \quad \forall j \in [N], t \in [T], v \in [V] \tag{6}$$

$$\psi_{jt}^{av} \leq d_{jt}^{av} \quad \forall j \in [N], t \in [T], a \in [A_\psi], v \in [V] \tag{7}$$

$$w_{jt}^{av} = \left(w_{j,t-1}^{a-1,v} - \psi_{j,t-1}^{a-1,v} \right) \quad \forall j \in [N], t \in [T+1], a \in [A_w] \setminus \{0\}, v \in [V] \tag{8}$$

$$\sum_{l,s \in [S_{lt}]} q_{ljs} = w_{jt}^{0v} \quad \forall j \in [N], t \in [T], v \in [V] \tag{9}$$

$$q_{ljs} \leq \frac{C_{lt}}{a_{lj}} y_{ljs} \quad \forall l \in [L], j \in [N], t \in [T], s \in [S_{lt}] \tag{10}$$

$$\sum_{i,j,s \in [S_{lt}]} \tau_{lij} z_{lijs} + \sum_{j,s \in [S_{lt}]} a_{lj} q_{ljs} \leq C_{lt} \quad \forall l \in [L], t \in [T] \tag{11}$$

$$\sum_j y_{ljs} = 1 \quad \forall l \in [L], t \in [T], s \in [S_{lt}] \tag{12}$$

$$q_{ljs} \geq m_{lj}(y_{ljs} - y_{lj,s-1}) \quad \forall l \in [L], j \in [N], t \in [T], s \in [S_{lt}] \tag{13}$$

$$z_{lijs} \geq y_{li,s-1} + y_{ljs} - 1 \quad \forall l \in [L], i, j \in [N], t \in [T], s \in [S_{lt}] \tag{14}$$

$$\psi_{jt}^{av},\ w_{jt}^{av},\ q_{ljs},\ z_{lijs} \geq 0;\ y_{ljs} \in \{0, 1\}. \tag{15}$$

Objective function (5) maximizes the profit over the planning horizon by subtracting the revenue of the sold products, the spoilage cost, sequence-dependent setup cost, and variable production cost.

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The quantity of spoiled products that reach the end of the shelf life without being sold is obtained in a straightforward manner with the proposed formulation since it corresponds to the inventory that reaches an age of u_j (it is given by $w_{jt}^{u_j,v}$).

Equation (6) does not allow the quantity of sold products of a given age to be above the demand curve derived from the customer willingness to pay. Equation (7) forces the sum of all sold products of different ages not to exceed the total demand for the product with the fresher state for each scenario. Hence, in this formulation, a strong assumption is made regarding the perfect control of the retailer over the inventory available to customers. The discussion about other inventory policies is out of the scope of this work. The readers are referred to Amorim et al. (2013) for more details.

Constraints (8) establish the inventory balance constraints that are modeled here in a propagation form that updates the age of the inventories throughout the planning horizon for each demand scenario and discounts the products sold. Equation (9) establishes that the production in a certain macroperiod on all lines for a given product is equal to the available stock of that same product with age 0 (maximum freshness). These constraints link the production planning with the demand fulfillment requirements and, thus, link the first- and second-stage decision variables of the stochastic model. Constraints (10) ensure that in order to produce a certain product, the necessary setup is performed. Moreover, at each moment only one product may be produced on a certain line (12) and each product lot is subject to a minimum lot size (13). Constraints (11) limit the use of the capacity with setups and production in each macroperiod and, finally, constraints (14) are responsible for tracking the changeover between products. Note that the integrality condition of variables z_{lijs} is not necessary.

4. Computational study

In this section, the performance of the hybrid path-relinking method is assessed through computational experiments on instances of the S-LSP-PP. To test the proposed method, we run a C++ implementation of the algorithm with the mixed-integer programming solver CPLEX 12.5.1 on a PC with an Intel Core i7-3770-3.40 GHz processor under a Microsoft Windows 7 platform.

4.1. Data generation

A total of 54 instances were systematically generated, following a methodology similar to the one proposed by Haase and Kimms (2000); therefore, L was set to 1. For all products $a_{lj} = 1$, $c_{lj} = 0.5$, $p_j = 2$, and $\bar{p}_j = 2$. Moreover, the machine is set up for product 1 at the beginning of the planning horizon. The number of products N is 5, 10, and 15. The number of macroperiods T is 5, 10, and 20. The number of microperiods within a macroperiod ($|S_{lt}|$) is set at the value of N allowing for all products to be produced in each macroperiod with minimum lot sizes (m_{lj}) of 1 unit. For the setup times between products (τ_{lij}), the interval [2,10] was used for the 15 products (except for the case where $i = j$, and the setup is 0). Shelf lives (u_j) were generated for all 15 products for each possible planning period length choosing randomly from the interval [1, T]. The setup

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cost s_{lij} for a changeover from product i to product j on line l is computed as $s_{lij} = 50\tau_{lij}$. Each element of the initial demand matrix (d_{jt}^{0v}) for the average scenario with 15 products (rows) and 20 macroperiods (columns) was randomly generated on the interval [40, 60]. Afterwards, using Equation (4) the complete demand for all ages is created (d_{jt}^{av}). For the possible demand realizations, five different scenarios were generated by multiplying the average demand elements by 0.6, 0.8, 1.2, and 1.4. We assume that every scenario has the same probability of occurrence and test two cases: (a) all five scenarios and (b) three scenarios only with the intermediate scenario and less extreme ones (multiplying the average demand by 0.8 and 1.2). Finally, the capacity per macroperiod C_{lt} is determined according to

$$C_{lt} = \frac{\sum_{j,v} d_{jt}^{0v}}{U|[V]|}, \quad \forall l, t,$$

where the capacity utilization U is equal to 0.7, 0.8, and 0.9. It is important to note that the utilization of capacity is a rough estimate, as setup times do not influence the computation of C_{lt} and an average value for demand is considered across all scenarios. In summary there are

$$|\{5, 10, 15\}| \times |\{5, 10, 20\}| \times |\{3, 5\}| \times |\{0.7, 0.8, 0.9\}| = 56 \text{ instances.}$$

4.2. Parameters setting

The proposed hybrid path-relinking method only requires a few parameters to be set. The effort was expended for solving each scenario with the branch-and-bound procedure, which was set using both the integrality gap and solving time criteria. Therefore, a scenario is solved only while either the integrality gap is greater than 20% or solving time is below 300 seconds. These parameters were set after preliminary results showed that besides finding fast feasible solutions for each scenario, it is relevant to feed near-optimal solutions for each scenario in order to find solutions that have a good performance in a stochastic setting with the hybrid path-relinking method. For the parameter ϵ, which is responsible for controlling the neighborhood exploration of each path-relinking iteration, its value is chosen in such a way that about 70% of the starting solution is converted into the guiding one.

4.3. Results

To assess the performance of our algorithm, we compared it to the solutions found for the DEM using CPLEX. The hybrid path-relinking was lasted with the specified parameters (Section 4.2) and CPLEX with its default parameters. Moreover, for both methods, a time limit of 3600 seconds was set.

Performance profiles are used (Dolan and More, 2002) in order to assess the performance of both approaches. These performance profiles allow a fair comparison of methods ($s \in S$) within

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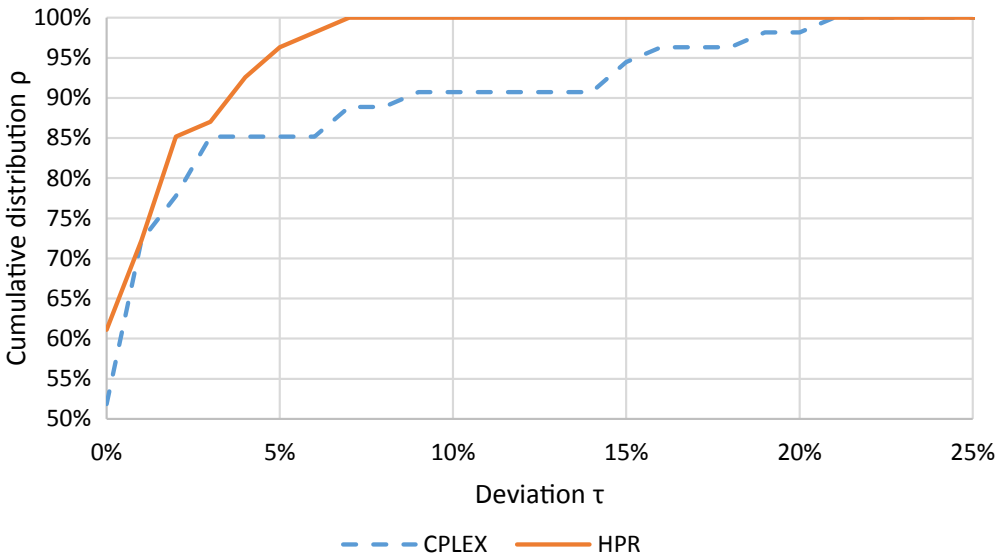


Fig. 3. Performance profile for CPLEX and HPR.

an instance set ($p \in P$) by using a performance measure (m), which is the value of the stochastic objective function. For each instance p and each method s, a performance ratio $r_{p,s}$ is obtained using the following expression:

$$r_{p,s} = \frac{\max\{m_{p,s'} : s' \in S\} - m_{p,s}}{\max\{m_{p,s'} : s' \in S\}}. \tag{16}$$

Having computed these values, for each method s the proportion of instances solved with a performance ratio less or equal than τ ($\rho_s(\tau)$) is obtained using the following expression (17):

$$\rho_s(\tau) = \frac{|p \in P : r_{p,s} \leq \tau|}{|P|}. \tag{17}$$

Figure 3 plots the performance profile of both CPLEX and HPR. The value $\rho_s(\tau)$ acts as a cumulative function and therefore the closer the curve to the top left curve the better it is.

Overall, the graph shows that the HPR performs better than CPLEX for the considered instances set. Its curve completely dominates the one related with CPLEX. In the worst case, the HPR has a deviation τ of around 6% while the deviation τ for CPLEX is about 20%.

The best-found solutions (BFSs) for each method and all instances are presented in Tables 1 and 2. For each instance, the best solution, running time, and relative difference between the solutions of both methods are reported. Moreover, averages for the running times and obtained improvement are also presented.

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Table 1
Results for the best-found solution of each method (instances with three scenarios)

Instance	Solution value		Running time (seconds)		Improvement (%)
$N \times T \times V \times U$	CPLEX	HPR	CPLEX	HPR	(HPR-CPLEX)/CPLEX
5 × 5 × 3 × 0.7	475.94	475.94	3600	934	0
5 × 10 × 3 × 0.7	1339.11	1339.11	3600	954	0
5 × 20 × 3 × 0.7	2370.14	2334.97	3600	2471	−1
10 × 5 × 3 × 0.7	1045.03	1045.03	3600	1172	0
10 × 10 × 3 × 0.7	2576.16	2576.16	3600	3068	0
10 × 20 × 3 × 0.7	4552.60	4482.30	3600	3600	−2
15 × 5 × 3 × 0.7	1342.97	1379.08	3600	2367	3
15 × 10 × 3 × 0.7	3408.90	3476.50	3600	3600	2
15 × 20 × 3 × 0.7	4418.02	5539.56	3600	3600	25
5 × 5 × 3 × 0.8	477.73	474.73	3600	920	−1
5 × 10 × 3 × 0.8	1301.97	1314.40	3600	966	1
5 × 20 × 3 × 0.8	2368.78	2289.23	3600	1872	−3
10 × 5 × 3 × 0.8	1052.15	1052.25	3600	1265	0
10 × 10 × 3 × 0.8	2544.56	2575.98	3600	3600	1
10 × 20 × 3 × 0.8	4441.84	4262.25	3600	3600	−4
15 × 5 × 3 × 0.8	1417.07	1459.90	3600	3600	3
15 × 10 × 3 × 0.8	3402.98	3298.49	3600	3600	−3
15 × 20 × 3 × 0.8	4403.11	5222.62	3600	3600	19
5 × 5 × 3 × 0.9	467.83	467.83	3600	982	0
5 × 10 × 3 × 0.9	1320.30	1325.61	3600	1323	0
5 × 20 × 3 × 0.9	2354.01	2314.70	3600	3061	−2
10 × 5 × 3 × 0.9	1040.80	1040.80	3600	1363	0
10 × 10 × 3 × 0.9	2583.06	2460.51	3600	3600	−5
10 × 20 × 3 × 0.9	4399.30	4143.65	3600	3600	−6
15 × 5 × 3 × 0.9	1379.63	1355.91	3600	3469	−2
15 × 10 × 3 × 0.9	3257.51	3061.00	3600	3600	−6
15 × 20 × 3 × 0.9	4309.61	4695.92	3600	3600	9
Average	–	–	3600	2569.89	1

These detailed results, disaggregated per instance, indicate that the proposed method outperforms CPLEX especially in the hardest instances (with 15 products and 20 periods). Moreover, the average of the improvement is higher for the instances with five scenarios. This suggests that by managing more candidate solutions, the HPR is able to incorporate the information of the individual scenarios and deliver a better result. For the smallest instances, both methods perform very similarly. However, these better results were found, on average, in a less amount of time.

Figure 4 plots the deviation to the BFS over the running time for both CPLEX and HPR. The graph shows that until around 300 seconds, CPLEX is obtaining better solutions than the HPR. This is mainly due to the warming period of the proposed method, when it is exploring the individual scenarios. After this period, the HPR outputs on average a solution that is closer to the BFS of a given instance.

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Table 2
Results for the best-found solution of each method (instances with five scenarios)

Instance	Solution value		Running time (seconds)		Improvement (%)
$N \times T \times V \times U$	CPLEX	HPR	CPLEX	HPR	(HPR-CPLEX)/CPLEX
5 × 5 × 5 × 0.7	1302.27	1305.56	3600	1609	0
5 × 10 × 5 × 0.7	3154.17	3105.36	3600	1736	−2
5 × 20 × 5 × 0.7	5769.23	5784.58	3600	3600	0
10 × 5 × 5 × 0.7	2739.27	2733.30	3600	2409	0
10 × 10 × 5 × 0.7	6114.01	5965.76	3600	3600	−2
10 × 20 × 5 × 0.7	11,080.02	11,068.47	3600	3600	0
15 × 5 × 5 × 0.7	3891.21	3958.42	3600	3600	2
15 × 10 × 5 × 0.7	8178.86	8776.06	3600	3600	7
15 × 20 × 5 × 0.7	12,933.11	15,078.69	3600	3600	17
5 × 5 × 5 × 0.8	1306.77	1305.56	3600	1647	0
5 × 10 × 5 × 0.8	3128.57	3131.04	3600	1551	0
5 × 20 × 5 × 0.8	5739.09	5765.77	3600	3600	0
10 × 5 × 5 × 0.8	2738.35	2738.86	3600	2345	0
10 × 10 × 5 × 0.8	6082.92	5994.21	3600	3600	−1
10 × 20 × 5 × 0.8	11,266.63	10,900.85	3600	3600	−3
15 × 5 × 5 × 0.8	3837.00	3940.28	3600	3600	3
15 × 10 × 5 × 0.8	8215.51	8255.34	3600	3600	0
15 × 20 × 5 × 0.8	12,800.18	14,904.89	3600	3600	16
5 × 5 × 5 × 0.9	1305.56	1305.56	3600	1647	0
5 × 10 × 5 × 0.9	3111.59	3141.44	3600	1699	1
5 × 20 × 5 × 0.9	5768.48	5769.47	3600	3600	0
10 × 5 × 5 × 0.9	2677.13	2738.86	3600	2345	2
10 × 10 × 5 × 0.9	6102.88	5994.21	3600	3600	−2
10 × 20 × 5 × 0.9	10,958.37	10,900.85	3600	3600	−1
15 × 5 × 5 × 0.9	3697.93	3940.28	3600	3600	7
15 × 10 × 5 × 0.9	8270.28	8255.34	3600	3600	0
15 × 20 × 5 × 0.9	12,183.20	14,904.89	3600	3600	22
Average	-	-	3600	3029.19	2

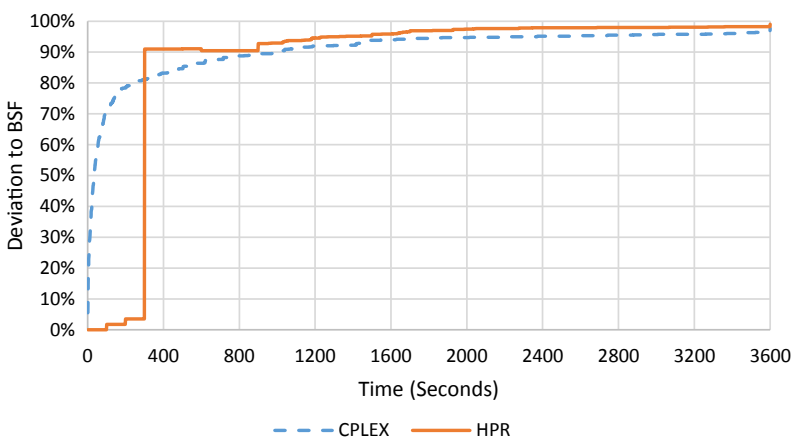


Fig. 4. Efficiency profile in terms of time for CPLEX and HPR.

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5. Conclusions and future work

This paper shows the suitability of hybrid methods to solve complex stochastic problems, such as the stochastic lot sizing and scheduling of perishable goods. Specifically, we propose a novel hybridization of a truncated path-relinking with a mixed-integer solver that takes advantages of the special structure of these type of problems. This method is innovative because the path-relinking method is applied in a completely different context than its original use. From a reasonable (good) upper bound for each demand scenario separately, a path-relinking method works on the most promising solutions from the stochastic point of view. Since we are dealing with a two-stage stochastic model with only linear second-stage variables, each iteration evaluation of the path-relinking is rather fast. Results show the increased advantage of this method over CPLEX alone especially for the hardest instances.

Future work is to be performed in three directions. Improve the decision on which starting and guiding solutions to use, reduce the need of the mixed-integer linear solver in order to improve scalability, and finally, assess the performance of this method under harder instances.

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